{logo here*} COMMUTATIVITY OF ADDITION {logo here*}

{* The logo is "p + d = d + p" as an animated .gif spinning about the = sign.}

{NOTE: The information in this smaller type enclosed in braces are instructions from the tutorial author to the Web page designer. To see how these instructions would be interpreted and implemented by the Web page designer, see the sample tutorial on our Web site. If you choose to submit a Web-ready tutorial, these instructions will not be necessary.}

PREREQUISITE: This tutorial assumes the reader is familiar with the four basic arithmetic operations of real numbers (addition, subtraction, multiplication and division) and with the concepts of absolute value and length.

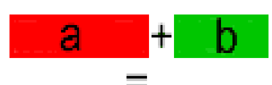
MATERIALS NEEDED: Construction paper, scissors, ruler.

SECTION 1

NOTE: If you have completed the construction paper exercise, go to Section 2.

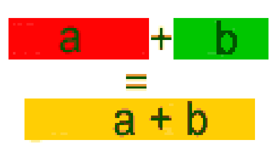
DEFINITION: An operation # on two numbers is **COMMUTATIVE** if *a* # *b* = *b* # *a* for all numbers *a* and *b*.

ADDITION: In this tutorial we deal only with commutativity of addition. We all know, of course, that 7 + 2 = 2 + 7 = 9, so addition certainly looks like it may be commutative; but how do we know that it works for all numbers. We will start our exploration with positive real numbers. Then numbers *a* and *b* can be represented as sticks of length *a* and *b* and addition can be represented by putting the sticks together end to end.

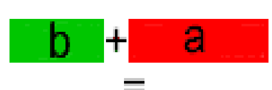


Click HERE to add *a* and *b*.

{NOTE: An animated .gif will convert the above figure to:}

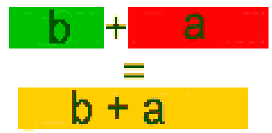


Now let's try it the other way.

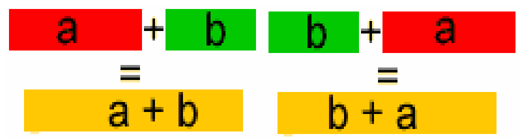


Click HERE to add *b* and *a*.

{NOTE: An animated .gif converts the above figure to:}

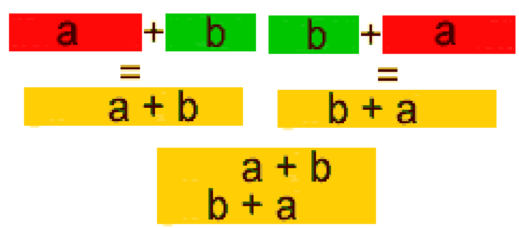


Now that we have both *a* + *b* and *b* + *a*, let's see if they are equal.

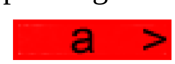


Click HERE to check for equality.

{NOTE: An animated .gif converts the above figure to:}



They sure look equal from here!

Expanding our investigation to include negative numbers gets a little trickier. If *a* is a real number we represent it with a stick whose length is the absolute value of *a*, but we add an arrow head pointing to the right if *a* is positive or to the left if *a* is negative. Thus, a positive *a* is  and a negative *a* is . Similarly, a positive *b* is  and a negative *b* is . We will call the end of the number bar with the arrow the **head** of the number and the other end the **tail**. The addition *a* + *b* can now be represented by placing the tail of *b* at the head of *a*. Then *a* + *b* is the bar with its tail at the tail of *a* and its head at the head of *b*. For example, if *a* is negative and *b* is positive, *a* + *b* is shown below.

As another example, if *a* and *b* are both negative, then *b* + *a* is as follows.

Using construction paper, make bars for positive and negative *a* and for positive and negative *b*. Now compute, as shown above, *a* + *b* and *b* + *a* for each of the four possible combinations: *a* and *b* both positive, both negative, *a* positive and *b* negative, and *a* negative and *b* positive. Verify that in each case, *a* + *b* = *b* + *a*.

This completes Section 1. When you have finished the work with the construction paper,

GO TO SECTION 2